Proc. Nat. Acad. Sci. USA
Vol. 68, No. 6, pp. 1192–1194, June 1971

A New Labor Theory of Value for Rational Planning Through Use of the Bourgeois Profit Rate

(golden-rule state/Marxian values/technical progress)

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Communicated March 31, 1971

ABSTRACT **To maximize steady-state per capita consumptions, goods should be valued at their "synchronized labor requirement costs", which are shown to deviate from Marx's schemata of "values" but to coincide with bourgeois prices calculated at dated labor requirements, marked-up by compound interest, at a profit or interest rate equal to the system's rate of exponential growth. With capitalists saving all their incomes for future profits, workers get all there is to get. Departures from such an exogenous, or endogenous, golden-rule state are the rule in history rather than the exception. In the case of exponential labor-augmenting change, it is shown that competitive prices will equal historically embodied labor content.**

MARKET RELATIONS

In an economic system where all goods are ultimately producible by labor, i.e., in which any one good is produced with direct labor and one or more of the goods in the system (including possibly itself), if the rate of profit or interest were always zero, the equilibrium competitive prices would be exactly *equal* to the *total* embodied labor required for each good. This accords with the views of Karl Marx, both in Volume I of Capital [1] and Volume III, and with the views of bourgeois economists such as Adam Smith, David Ricardo, Leon Walras, and Wassily Leontief.

If, however, there is a positive interest or profit rate, labor will not receive a real wage large enough to buy all the consumption goods producible by labor in the stationary synchronized equilibrium. Bourgeois economists, and Marx in the posthumous Volume III, demonstrate that with positive interest the prices will no longer be proportional to the respective total embodied labor contents. Thus, if the same historic labor total, say 1 labor, is needed for either a liter of grape juice or for a liter of wine, but for wine the labor is needed 2 time-units earlier rather than only 1 time-unit earlier as for grape juice, the ratio of wine price to grape juice price will not be $P_2/P_1 = 1/1$, but will instead vary with the profit rate per period r, being $P_2/P_1 = 1(1+r)^2/1(1+r) = 1+r$. In terms of the familiar Leontief notation [2], in which $a = [a_{ij}]$ denotes input needed of good i to produce a unit of good j and $a_0 = [a_{0j}]$ denotes the corresponding amount of direct labor input, we have in matrix terms for bourgeois prices

$$\begin{aligned}[P_1,\ldots,P_n] &= a_0(1+r)[I - a(1+r)]^{-1} \\ &= [A_{01}(r),\ldots,A_{0n}(r)], A_{0i}'(r) > 0. \qquad (1)\end{aligned}$$

In volume I, Marx hoped to break new ground by insisting that all "profit" or "surplus" be reckoned at each stage of production as a mark-up on direct labor alone. Hence, Marx's "values" systematically "contradict" bourgeois "prices" and do remain proportional to embodied labor—but no longer equal to such labor contents because of the mark-ups. In matrix terms, values $[\pi_i]$ are defined by

$$\begin{aligned}[\pi_1,\ldots,\pi_n] &= a_0[I - a]^{-1}(1+s) \\ &= [A_{0j}(0)\ (1+s)] \qquad (2)\end{aligned}$$

where s is the "rate of surplus value" or mark-up. Note that for *all* s, we get π_i/π_j equal to the P_i/P_j calculated for $r = 0$ and representing total embodied labor requirements.

Clearly $P_i/P_j \neq \pi_i/\pi_j$ since, in general,

$$A_{0i}(r)/A_{0j}(r) \neq A_{0i}(0)/A_{0j}(0).$$

Thus, grape juice and wine have equal "values" since they both involve unit labor inputs; but their bourgeois "prices" differ from the Marxian values because the former calculate labor requirements, *dated* by when they occur and carried forward at nefarious compound interest.

OPTIMAL PLANNING RELATIONS

Suppose a technocratic computer allocates resources in a population growing exponentially, so that total labor supply is given by

$$L(t) = L^0(1+g)^t, \qquad g \geq 0 \qquad (3)$$

Suppose any one good, say Q_j, is alone to be produced for consumption, presumably by workers. The maximum amount of this good available for steady-state per capita consumption can be easily demonstrated also to grow at the exponential rate $(1+g)$ or by

$$C_j(t) = C_j^0(1+g)^t \qquad (4)$$

Let us define $L(t)/C_j(t)$ as the "synchronized needed labor cost" of good j, $\lambda_j(g)$, or

$$\lambda_j(g) = L(t)/C_j(t) = L^0/C^0 \qquad (5)$$

Then surely in a rational planned society, where class exploitation is abolished, all goods should be "valued" or "priced" (the terms may now be used interchangeably) at their true "synchronized needed labor costs". Thus, along and in contrast with bourgeois prices of (1) and Marxian values of (2), we have "rational values or prices"

$$[P_1^*,\ldots,P_n^*] = [\lambda_1(g),\ldots,\lambda_n(g)]. \qquad (6)$$

IDENTITY OF RATIONAL VALUES AND BOURGEOIS-CALCULATED PRICES

We now state a fundamental theorem [3] that in a sense brings into discount the worthwhileness for planning of Marx's innovations in Volume I of *Capital.*

THEOREM. Rational values for optimal planning in synchronized exponential growth, namely "synchronized labor requirements costs" of each good, are identically equal to bourgeois costs of production calculated by marking up dated labor with a golden-rule profit or interest rate equal to the system's rate of exponential growth g.

$$\text{I.e., } [\lambda_1(g),\ldots,\lambda_n(g)] \equiv [A_{01}(g),\ldots,A_{0n}(g)] \not\equiv [\pi_1,\ldots,\pi_n] = [A_{0j}(0)(1+s)].$$

To prove this basic theorem, we have only to calculate explicitly the respective synchronized labor requirement costs for producing the respective goods.

Thus, consider the grape juice–wine example. Only 1 unit of direct labor is needed, but for wine it is needed two periods earlier, when we have only $L(t)/(1+g)^2$ available. In contrast, for grape juice the required labor is needed only one period ago, when we have $L(t)/(1+g)$ available. It is obvious, then, that the ratio of steady-state wine consumable to grape juice consumable is $1/(1+g)$, so that wine is "rationally dearer" than grape juice by the factor $1+g$. Thus, g, the rate of growth, does act like a fictitious profit rate that *must* be applied for all rational planning.

To prove the theorem in the general case, we write down the allocations of total labor and total *gross* productions of all goods—which will generally all be positive even though only a specified good j is to be consumed net by final consumers, because intermediate inputs of all goods will generally be needed somewhere along the line to produce net C_j:

$$\begin{aligned} L(t) &= L_1(t) + \ldots + L_n(t) \\ &= a_{01}Q_1(t+1) + \ldots + a_{0n}Q_n(t) \qquad (7) \\ Q_i(t) &= Q_{i1}(t) + \ldots + Q_{in}(t) + \delta_{ij}C_j(t) \\ &= a_{i1}Q_1(t+1) + \ldots + a_{in}Q_n(t+1) + \delta_{ij}C_j(t), \end{aligned}$$

where $\delta_{ij} = 1$ when $i = j$ and zero otherwise. Note that inputs at time t produce outputs at one period later, $t+1$. Now verify that all totals, $Q_j(t)$, grow at the exponential rate $(1+g)$, according to

$$Q_i(t) = {}_jQ_i^0(1+g)^t = Q_i(t+1)/(1+g), \qquad (8)$$

where this newly-defined coefficient, ${}_jQ_i^0$, depends on g, on which good is chosen for consumption (as shown by the prefix j), and on the $[a_{0j}, a_{ij}]$ technology of the system. These constants must satisfy (7), or

$$\begin{aligned} L^0 &= \sum_{k=1}^{n} a_{0k}(1+g)\,{}_jQ_k^0 \\ {}_jQ_i^0 &= \sum_{k=1}^{n} a_{ik}(1+g)\,{}_jQ_k^0 + \delta_{ij}L_0/\lambda_j(g) \qquad (9) \end{aligned}$$

Note that use has been made here of the definition in (5) of $\lambda_j(g)$. Solving in matrix terms, we have

$$\begin{aligned} [{}_jQ_i^0] &= L_0/\lambda_j(g) \text{ times } j\text{th column of } [I - a(1+g)]^{-1} \\ L_0 &= L_0/\lambda_j(g) \text{ times } a_0(1+g) \text{ times } j\text{th column of } [I - a(1+g)]^{-1} \qquad (10) \end{aligned}$$

Hence,

$$\begin{aligned} [\lambda_j(g)] &= a_0(1+g)\,[I - a(1+g)]^{-1} \text{ column for column} \qquad (11) \\ &\equiv A_0(g) = [A_{0j}(g)] \end{aligned}$$

by the definition of (1) provided the profit rate there, r, is set equal to the system's growth rate, g. Q.e.d.

Synchronized labor costs, as defined here, are seen to be interpretable as the ordinary embodied labor requirements for a fictitious system in which every $[a_{0j}, a_{ij}]$ coefficient of the actual system is pretended to be blown up by the growth factor $(1+g)$. The common sense of this blow-up rests in recognition of the fact that, for a rapidly growing system, the actual labor is required at an earlier date; hence, when we relate the C_j producible to the current *higher* labor, we do find enhanced Labor/Consumption (L/C) ratios.

LABOR-AUGMENTING INVENTION

A second case in which the bourgeois price relations of (1) and (2) will hold is provided by an economy in which the direct labor requirements decline exponentially as a result of technological change, i.e., in which $[a_{0j}(t)] = [a_{0j}(1+\gamma)^{-t}]$. If the planners in such a dynamic economy insist upon costing goods out so that there is no surplus over and above wage and raw material outlays—and if they wish to be in the highest growth path with price ratios expressing the relative availability of goods, they must so price—then equilibrium prices are proportional to Marxian values. Prices reflect the amount of labor time "necessary to produce the goods" (in the sense of past labor actually historically "embodied" in the current goods). The equilibrium price relations become, in matrix terms,

$$P(t) = [W(t-1)a_0(1+\gamma)^{-t} + P(t-1)a]\,(1+0) \qquad (12)$$

Setting

$$W(t) \equiv 1, \qquad P(t-1) = [p_j(1+\gamma)^{-t}] = (1+\gamma)^{-1}P(t)$$

defines the equilibrium row vector $(1+\gamma)^tP(t-1)/W(t) = p$,

$$\begin{aligned} p &= a_0(1+\gamma)\,[I - a(1+\gamma)]^{-1} \qquad (13) \\ &= A_0(\gamma) = [A_{01}(\gamma),\ldots,A_{0n}(\gamma)] \end{aligned}$$

This proves a second dynamic theorem already enunciated by one of us [4].

THEOREM. A systems ubject to labor-augmenting technical change at percentage rate γ will have its zero mark-up cost ratios, and hence values in the orthodox Marxian sense, that are proportional to bourgeois prices calculated at a profit rate precisely equal to the rate of technical change.

The two theorems come together in the case where the natural rate of growth of labor, g, is compounded with labor-augmenting growth, γ, so that total labor in "efficiency units" grows like $1 + G = (1+g)(1+\gamma)$. Then, rationally-planned price ratios will be formed from ratios of the $A_0(G)$ elements, whose G must be the rate of profit used in all price-ratio calculations. Charging these prices will give the highest possible consumption to labor consistent with financing the widening of capital or net investment needed to keep every intermediate good growing at the system's exponential rate G. That highest possible consumption involves growth of the real wage at the rate of labor-productivity growth, γ.

In connection with the second dynamic theorem, one is warned that with rising real wages the proportions in which goods are consumed will presumably change, away from necessities and toward luxuries. When proportions change, golden-age states of steady-state equilibrium lose relevance, and relations like (1), (6), or (13) have to reckon with capital-gains effects as determined in optimal-control models.

If, along with the reductions in the a_{0j} direct-labor requirements, the a_{ij} intermediate coefficients are also lowered by technical progress, approaching asymptotically, as $t \rightarrow \infty$, limiting values, a_{ij}^*, then (13) will hold asymptotically in terms of a^*. If the a_{ij} were exponentially declining without limit, relative prices would ultimately approach those of direct-labor requirements alone.

CONCLUSION

The truth of these basic theorems will come as no surprise to those familiar with the golden-rule theories [5]. However, a succinct statement and demonstration of the *technocratic* interpretation of "labor costs" in such a state should be useful. Of course the present g is not necessarily an exogenous parameter, but might depend on the excess of real-wages over some specifiable subsistence notion. If capitalists always accumulate all—Marx's "Law of Moses and the Prophets"—g^* will get positivistically determined (i.e., by laissez-faire interactions, without planning) at that rate which, when it is applied as a positive profit that reduces the real wage from all that labor can truly produce, just evokes a rate of population increase equal to itself.

Such a theory, which is a generalization of Marx's exploitation theory, need in principle encounter no eventual breakdown or realization crisis [6]. It has refutable or verifiable consequences, such as that technical improvements will raise (*a*) the rate of profit, (*b*) the rate of growth of the system, and (*c*) the real wage. (The same consequences follow even if capitalists and workers each save any constant proportions of their respective incomes, but then the interest rate may well exceed the growth rate.) But to accord with the facts of recent centuries, we must modify the view that (*i*) population growth is a simple, rising function of the real-wage level, that (*ii*) capitalists save all with workers saving nothing, and (perhaps) modify the view that (*iii*) nothing can be hypothesized about the character of technical change in terms of its probable effects on wages and profits. Limitations of nonproducible natural resources should also be recognized.

* We acknowledge aid from the Ford Foundation permitting a (1971) period of collaboration at M.I.T., financial aid from the National Science Foundation, and editorial assistance from Mrs. Jillian Pappas.

1. Marx, K., *Capital*, Vol. I (1867), Vol. III (1894).
2. Samuelson, P., *Proc. Nat. Acad. Sci. USA*, **67**, 423–425 (1970), gives notations and references.
3. (See) von Weizsäcker, C. C., forthcoming lecture notes on capital theory (Springer Verlag, Heidelberg, 1971), and Samuelson, P., "The Marxian Notion of Exploitation: the So-called Transformation Problem", to be published in the *Journal of Economic Literature*, 1971.
4. von Weizsäcker, C. C., "Bemerkungen zu einem Symposium über Wachstums—theorie und Produktions funktionen," *Kyklos*, **16**, 438–457 (1963).
5. (See) Phelps, E., *Golden Rules of Economic Growth: Studies of Efficient and Optimal Investment* (Norton, New York, 1967), for references.
6. (See) Sweezy, P., *Theory of Capitalist Development* (Oxford University Press, New York, 1942), for references to Marx, Rosa Luxemburg, *et al.*